

EE105 – Fall 2014

Microelectronic Devices and Circuits

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Lecture11-Cut-off Freq, Transit Time,
Early Voltage, Bias

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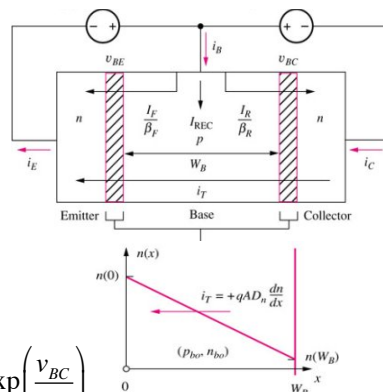


Minority Carrier Transport in the Base Region

- BJT current dominated by **diffusion of minority carriers** (electrons in npn and holes in pnp transistors) **across base region**.
- Base current consists of hole injection back into emitter and collector and a small additional current to replenish holes lost to recombination with electrons in base.
- Minority carrier concentrations at the two ends of the base region are:

$$n(0) = n_{bo} \exp\left(\frac{v_{BE}}{V_T}\right) \quad \text{and} \quad n(W_B) = n_{bo} \exp\left(\frac{v_{BC}}{V_T}\right)$$

n_{bo} is equilibrium electron density in the p -type base region.



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Minority Carrier Transport in Base for npn

- For narrow base devices, minority carrier density decreases linearly across the base, and the diffusion current in the base is:

$$I_C \approx qAD_n \frac{dn}{dx} = qAD_n \frac{n(0) - n(W_B)}{W_B} = \frac{qAD_n}{W_B} n_{b0} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow I_S = qAD_n \frac{n_{b0}}{W_B} = \frac{qAD_n}{W_B} \frac{n_i^2}{N_{AB}}$$

A : emitter area

D_n : electron diffusion coefficient

W_B : base width

$n_{b0} = \frac{n_i^2}{N_{AB}}$: equilibrium electron concentration in base (minority carrier)

N_{AB} : acceptor doping concentration in base

n_i : intrinsic carrier concentration ($=10^{10} \text{ cm}^{-3}$ for Si)

Mass-action law: $n \cdot p = n_i^2$ at equilibrium



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Minority Carrier Transport in Base for pnp

- Saturation current for the pnp transistor is

$$I_S = qAD_p \frac{p_{b0}}{W_B} = \frac{qAD_p}{W_B} \frac{n_i^2}{N_{DB}}$$

A : emitter area

D_p : electron diffusion coefficient

W_B : base width

$p_{b0} = \frac{n_i^2}{N_{DB}}$: electron concentration in base (minority carrier)

N_{DB} : donor concentration in base

n_i : intrinsic carrier concentration ($=10^{10} \text{ cm}^{-3}$ for Si)

- Due to higher mobility of electrons than holes, the npn transistor conducts higher current than the pnp for a given set of applied voltages.

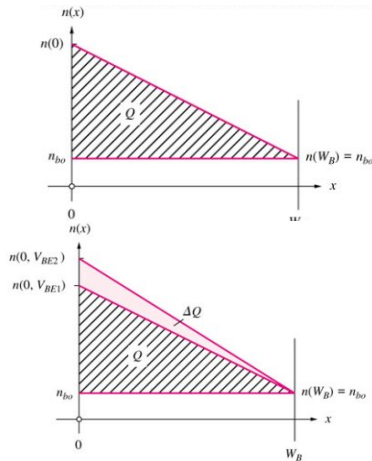


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Base Transit Time



- Forward transit time τ_F is the time constant associated with storing minority-carrier charge Q in base

$$Q = qAn_{bo} \left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right] \frac{W_B}{2}$$

$$i_T = \frac{qAD_n}{W_B} n_{bo} \left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right]$$

$$\tau_F = \frac{Q}{i_T} = \frac{W_B^2}{2D_n} = \frac{W_B^2}{2\mu_n V_T}$$

Note: D_n and μ_n are related by

$$\text{Einstein relation: } \frac{D_n}{\mu_n} = \frac{kT}{q} = V_T$$

- Transit time places upper limit on useful operating frequency of transistor.



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Diffusion Capacitance

- For v_{BE} and hence i_C to change, charge stored in base region must also change.
- Diffusion capacitance in parallel with forward-biased base-emitter diode models the change in charge with v_{BE} .

$$C_D = \left. \frac{dQ}{dv_{BE}} \right|_{Q-pt} = \frac{1}{V_T} \frac{qAn_{bo}W_B}{2} \exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_T}{V_T} \tau_F$$

- Since transport current normally represents collector current in forward-active region,

$$C_D = \frac{I_C}{V_T} \tau_F$$



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Cutoff-Frequency, Transconductance and Transit Time

- Forward-biased diffusion and reverse-biased pn junction capacitances of the BJT cause current gain to be frequency-dependent.
- Unity gain frequency f_T (or gain-bandwidth product):

$$\beta(f) = \frac{\beta_F}{\sqrt{1 + \left(\frac{f}{f_B}\right)^2}} \quad \text{where} \quad f_B = \frac{f_T}{\beta_F} \text{ is the gain cutoff-frequency}$$

- Transconductance is defined by:

$$g_m = \left. \frac{di_C}{dv_{BE}} \right|_{Q-Pt} = \frac{d}{dv_{BE}} \left[I_S \exp\left(\frac{v_{BE}}{V_T}\right) \right] \bigg|_{Q-Pt} = \frac{I_C}{V_T}$$

- Transit time is given by:

$$\tau_F = \frac{C_D}{g_m} \Rightarrow \omega_T = 2\pi f_T = \frac{1}{\tau_F} = \frac{g_m}{C_D}$$

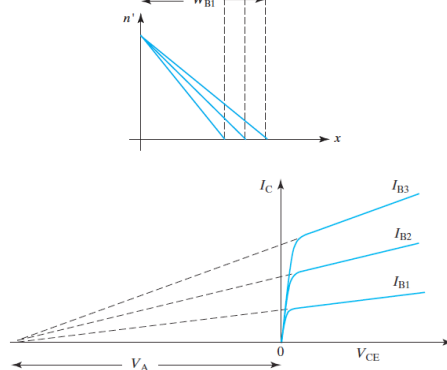
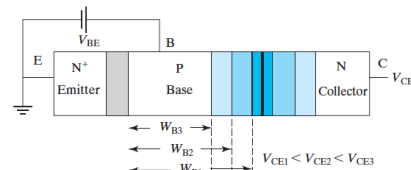


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Early Effect and Early Voltage



$$i_C = \frac{qAD_n}{W_B} n_{bo} \left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right]$$

As v_{CE} increases,

BC junction is more reverse-biased

\Rightarrow BC junction depletion width increases

\Rightarrow Base width decreases

$\Rightarrow i_C$ increases since $i_C \propto \frac{1}{W_B}$

This can be modeled by Early voltage:

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \left[1 + \frac{v_{CE}}{V_A} \right]$$

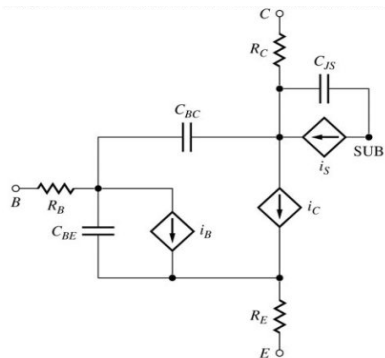


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BJT SPICE Model



- Besides capacitances associated with the physical structure, additional components are:
 - diode current i_S and substrate capacitance C_{JS} related to the large area pn junction that isolates the collector from the substrate and one transistor from the next.
- R_B is resistance between external base contact and intrinsic base region.
- Collector current must pass through R_C on its way to the active region of the collector-base junction.
- R_E models any extrinsic emitter resistance in device.



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Typical Values of BJT SPICE Model Parameters

- Saturation Current $I_S = 3 \times 10^{-17} \text{ A}$
- Forward current gain $\beta_F = 100$
- Reverse current gain $\beta_R = 0.5$
- Forward Early voltage $V_{AF} = 75 \text{ V}$
- Base resistance $R_B = 250 \text{ } \Omega$
- Collector Resistance $R_C = 50 \text{ } \Omega$
- Emitter Resistance $R_E = 1 \text{ } \Omega$
- Forward transit time $T_T = 0.15 \text{ ns}$
- Reverse transit time $T_R = 15 \text{ ns}$

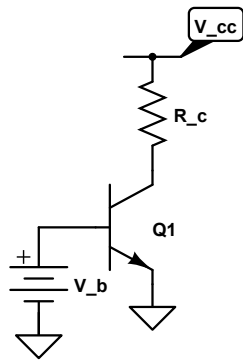


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BJT Bias: (1) Voltage Source to Base



$$I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

I_C varies exponentially with v_{BE} and V_T

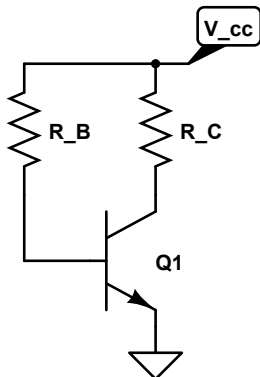
Very sensitive to bias voltages and temperature

Also sensitive to I_S

Not desirable!



BJT Bias: (2) Bias with V_{CC} and Resistor



$$I_B = \frac{V_{CC} - v_{BE}}{R_B}$$

$$I_C = \beta I_B = \frac{V_{CC} - v_{BE}}{R_B / \beta}$$

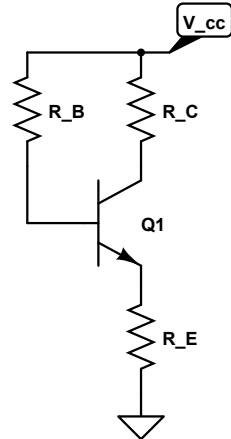
Since $V_{CC} \gg v_{BE}$, I_C is less sensitive to v_{BE}

It's also not sensitive to V_T or I_S

However, it is very sensitive to β



BJT Bias: (3) Bias with V_{CC} and R_B Plus R_E



KVL:

$$V_{CC} = I_B R_B + v_{BE} + I_E R_E$$

$$= \frac{I_C}{\beta} R_B + v_{BE} + \frac{I_C}{\alpha} R_E$$

$$I_C = \frac{V_{CC} - v_{BE}}{\frac{R_B}{\beta} + \frac{R_E}{\alpha}}$$

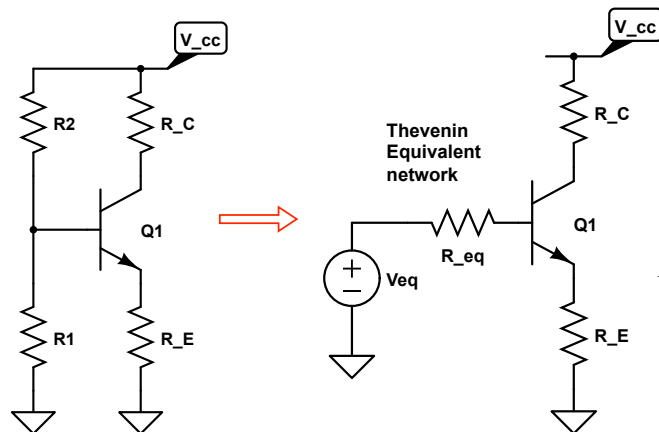
Since $V_{CC} \gg v_{BE}$, I_C is not sensitive to v_{BE}

$$\frac{R_B}{\beta} \ll \frac{R_E}{\alpha}, \text{ It's not sensitive to } \beta$$

Both are good.



BJT Bias: (4) Four-Resistor Bias



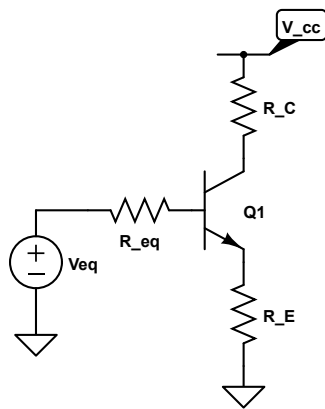
Thevenin
Equivalent
network

$$V_{EQ} = V_{CC} \frac{R_1}{R_1 + R_2}$$

$$R_{EQ} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



BJT Bias: (4) Four-Resistor Bias



$$I_C = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta} + \frac{R_E}{\alpha}}$$

Make $V_{EQ} \gg V_{BE}$, Insensitive to V_{BE}

$$\frac{R_B}{\beta} \ll \frac{R_E}{\alpha}, \text{ Insensitive to } \beta$$

Also, choose R_1 and R_2 such that $I_1, I_2 \gg I_B$

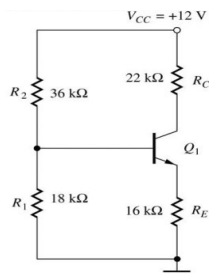
Then $I_1 \approx I_2$

The bias point (Q point) is independent of base current as well as current gain!

For design \Rightarrow choose $I_1 \approx I_2 = 10I_B$



Four-Resistor Bias Example

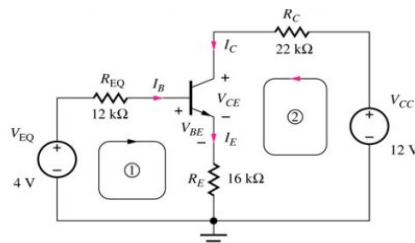


Thevenin Equivalent of
Base Bias Network

$$V_{EQ} = V_{CC} \frac{R_1}{R_1 + R_2} = 4 \text{ V}$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = 12 \text{ k}\Omega$$

$$\beta_F = 75$$



$$V_{EQ} = I_B R_{EQ} + V_{BE} + I_E R_E$$

$$I_B = \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1) R_E}$$

$$I_B = \frac{4 \text{ V} - 0.7 \text{ V}}{12 \text{ k}\Omega + (76) 16 \text{ k}\Omega} = 2.69 \text{ }\mu\text{A}$$

$$I_C = \beta_F I_B = 202 \text{ }\mu\text{A}$$

$$I_E = (\beta_F + 1) I_B = 204 \text{ }\mu\text{A}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 4.29 \text{ V}$$

Forward active region is correct

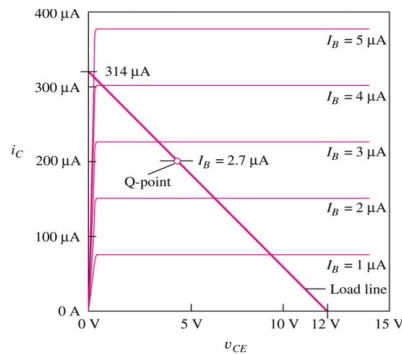
Q-point is (202 μA , 4.29 V)



Four-Resistor Bias Load-Line

- Load-line for the circuit is:

$$V_{CE} = V_{CC} - \left(R_C + \frac{R_E}{\alpha_F} \right) I_C = 12 - 38200 I_C$$



The two points needed to plot the load line are (0, 12 V) and (314 μA , 0). The resulting load line is plotted on the common-emitter output characteristics.

$I_B = 2.7 \mu A$ - the intersection of the corresponding characteristic with load line gives the Q-point: (200 μA , 4.3 V)



Four-Resistor Bias Design Guidelines

- Choose Thévenin equivalent base voltage

$$\frac{V_{CC}}{4} \leq V_{EQ} \leq \frac{V_{CC}}{2}$$

- Select R_1 to set $I_1 = 9 I_B$. $R_1 = \frac{V_{EQ}}{9 I_B}$

- Select R_2 to set $I_2 = 10 I_B$. $R_2 = \frac{V_{CC} - V_{EQ}}{10 I_B}$

- R_E is determined by V_{EQ} and the desired I_C . $R_E \cong \frac{V_{EQ} - V_{BE}}{I_C}$

- R_C is determined by desired V_{CE} . $R_C \cong \frac{V_{CC} - V_{CE} - R_E I_C}{I_C}$



Four-Resistor Bias for BJT Design Example

- **Problem:** Design 4-resistor bias circuit with given parameters.
- **Given data:** $I_C = 750 \mu\text{A}$, $\beta_F = 100$, $V_{CC} = 15 \text{ V}$, $V_{CE} = 5 \text{ V}$
- **Assumptions:** Forward-active operation region, $V_{BE} = 0.7 \text{ V}$
- **Analysis:** Divide $(V_{CC} - V_{CE})$ equally between R_E and R_C . Thus, $V_E = 5 \text{ V}$ and $V_C = 10 \text{ V}$; Choose nearest 5% resistor values

$$R_C = \frac{V_{CC} - V_C}{I_C} = 6.67 \text{ k}\Omega \rightarrow 6.8 \text{ k}\Omega \quad I_2 = 10I_B = 75.0 \mu\text{A}$$

$$R_E = \frac{V_E}{I_E} = 6.60 \text{ k}\Omega \rightarrow 6.8 \text{ k}\Omega \quad I_2 = 9I_B = 67.5 \mu\text{A}$$

$$V_B = V_E + V_{BE} = 5.7 \text{ V}$$

$$R_1 = \frac{V_B}{9I_B} = 84.4 \text{ k}\Omega \rightarrow 82 \text{ k}\Omega$$

$$I_B = \frac{I_C}{\beta_F} = 7.5 \mu\text{A}$$

$$R_2 = \frac{V_{CC} - V_B}{10I_B} = 124 \text{ k}\Omega \rightarrow 120 \text{ k}\Omega$$

